Calculation Policy

At Harrow Primary we strive to enable children to become fully numerate young people who not only have confidence in mathematics but are developing real enjoyment for the subject. Many methods are available to teach children and through this policy we intend to provide clarity on how we believe children will best understand the process of calculation in addition, subtraction, multiplication and division.

Oct, 2016
v3.0

INTRODUCTION

At Harrow Primary we strive to enable children to become fully numerate young people who not only have confidence in mathematics but are developing real enjoyment for the subject. Many methods are available to teach children and through this policy we intend to provide clarity on how we believe children will best understand the process of calculation in addition, subtraction, multiplication and division.

AIMS AND OBJECTIVES

- enable our children to become confident, numerate young people
- provide clarity to teachers, teaching assistants and parents
- simplify the process and methods of calculation
- provide a clear link between written and mental calculation

This policy aims to identify the progression in each of the four operations that the children will follow. Each stage is built upon previous experiences and all stages are levelled according to National Curriculum Levels to enable teachers to teach the most appropriate methods of written calculation for their children.

GUIDANCE NOTES

This policy contains guidelines upon which to base the modelling of written calculation in the relevant year groups. It is not expected that teachers simply teach from this document without first establishing through assessment at what stages the children are at. Children should not be made to go onto the next stage if they are not ready or they are not confident. For the purposes of differentiation, methods from years above or below can be used.

- Children are encouraged to approximate or estimate their answers before calculating.
- Children are encouraged to check their answers after calculation using an appropriate (inverse) strategy.
- Skills for estimating, rounding and checking using inverse operations will be taught as ongoing skills and will be built into plans.
- Children are encouraged to consider if a mental calculation would be appropriate before using written methods.
- The use of maths equipment and a practical, creative approach to teaching maths is encouraged throughout all year groups.

PROGRESSION OF MENTAL SKILLS

- 1:1 correspondence.
- Counting forwards and backwards in 1's from any number.
- Say 1 more/1 less with numbers up to 10,20.
- Number bonds to 5, 10 and 20.
- Doubling/halving up to 10, 20.
- Counting forwards and backwards in 2's, 5's and 10's from any number.
- Estimating numbers up to 100.
- Number facts up to 20.
- Understanding of place value and partitioning mentally.
- Rounding to the nearest 10, 100, 1000.
- Times tables - 2, 5, 10, 3, 6, 12, 4, 8, 9, 11, 7.
- Inverse operations between + and - and × and ÷.
- Near multiples to 100.
- Number bonds to 100.
- Adjusting by 1 when using adding 9 and 11 etc.

MONITORING OF THIS POLICY

The head teacher and the Key stage leaders will review this policy annually and update, modify or amend it as it considers necessary to ensure the policy meets the needs of Harrow Primary School.

DATE PUBLISHED: OCTOBER 2016

REVIEW DATE: OCTOBER 2017

APPROVED BY: ALIMUDDIN SHAIKH (HEADTEACHER)

ADDITION

PROGRESSION THROUGH CALCULATION

MENTAL CALCULATIONS (ONGOING)

These are a selection of mental calculation strategies:

1. Mental recall of number bonds

6 + 4 = 10_ + 3 = 10

25 + 75 = 100 19 + _ = 20

2. Mental recall of doubles

Use near doubles

6 + 7 = double 6 + 1 = 13

3. Addition using partitioning and recombining

34 + 45 = (30 + 40) + (4 + 5) = 79

4. Counting on or back in repeated steps of 1, 10, 100, 1000

86 + 57 = 143 (by counting on in tens and then in ones)

460 - 300 = 160 (by counting back in hundreds)

5. Add the nearest multiple of 10, 100 and 1000 and adjust

24 + 19 = 24 + 20 – 1 = 43

458 + 71 = 458 + 70 + 1 = 529

6. Use the relationship between addition and subtraction

36 + 19 = 55 19 + 36 = 55

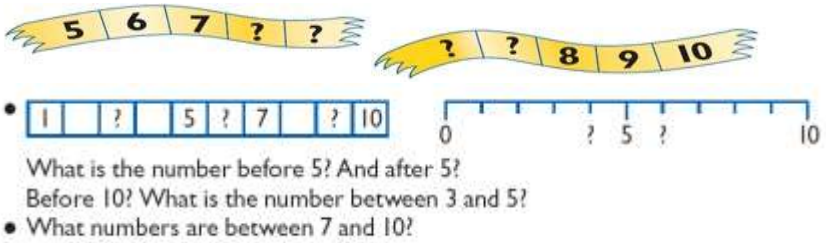
55 – 19 = 36 55 – 36 = 19

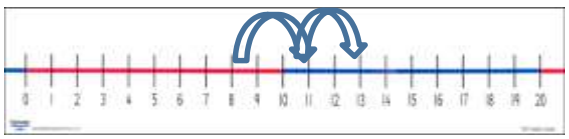
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

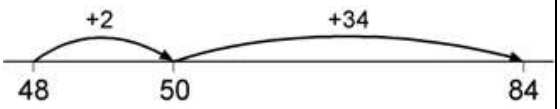
Points to remember:

Use the language 'calculation' not 'sum' ('sum' means 'plus or 'total'.)

WRITTEN METHODS FOR ADDITION

Phase 1 Develop secure one-one correspondence and understanding of addition	**Count accurately 0-10** • **Recognise and write numerals 1-10**  What is the number before 5? And after 5? Before 10? What is the number between 3 and 5? • What numbers are between 7 and 10? Count and add together sets of real objects and pictures. 3 + 2 = 5 

Phase 2: **The number line and 100 square** The number line helps children to move from using concrete objects. Children begin to split a number to add to the nearest multiple of 10 and then count on. The 100 square supports children's	**Phase 2** To use a number line to add one or more numbers together.  To be able to add through 10. 8 + 5 = 13  <u>100 Square</u> • To be able to add 10 to any number up to 100 using a 100 square by initially counting on 10.	**Key vocabulary** Add, more, count on, plus, sum, total, altogether, partition, how many. Multiple of 10,number line, 100 square

understanding when adding ten to any number the units stay the same and the tens go up. Eventually this will be done mentally.	9+10 = 10+9 = 19 To be able to add multiples of 10 to any number up to 100 using a 100 square. 34+40 = 74	
Phase 3: **The empty number line** • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens. • Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	**Phase 3** Steps in addition can be recorded on an empty number line. The steps often bridge through a multiple of 10. 8 + 7 =15  To be able to add 2 two digit numbers on an empty number line. 48 + 36 = 84  Or 	
Phase 4:	**Phase 4** Record steps in addition using partitioning: This	

Partitioning • The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums.	method can be used to support mental strategies. 47 + 76 = 40 +70 + 6 = 117 + 6 = 123 47 + 76 40 + 70 = 110 7 + 6 = 13 110 + 13 = 113	
Phase 5: **Column method This should be aimed to be taught from Year 4 onwards** • In this method, recording is reduced further. Carry digits are recorded above the numbers, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	**Phase 5** 1 11 11 47 258 366 +76 + 87 +458 123 345 824 Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.	**As above** Carry,

Phase 6: **Column method expanded to decimals** In this phase the standard method is expanded to include the use of decimals.	**Phase 6** 1 1 45.24 +26.59 71.83	

SUBTRACTION

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS (ONGOING)

These are a selection of mental calculation strategies:

1. Mental recall of addition and subtraction facts

10 – 6 = 4 17 - _ = 11
20 - 17 = 3 10 - _ = 2

2. Find a small difference by counting up

82 – 79 = 3

3. Counting on or back in repeated steps of 1, 10, 100, 1000

86 - 52 = 34 (by counting back in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

4. Subtract the nearest multiple of 10, 100 and 1000 and adjust

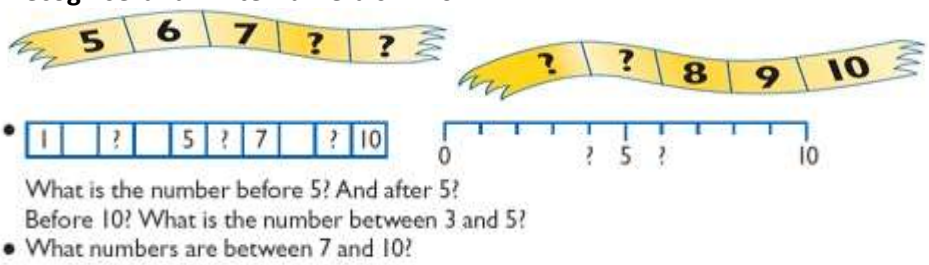
24 - 19 = 24 - 20 + 1 = 5
458 - 71 = 458 - 70 - 1 = 387

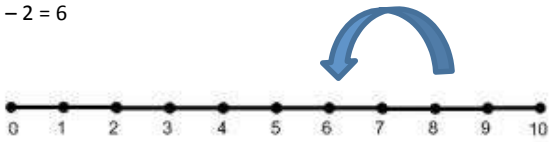
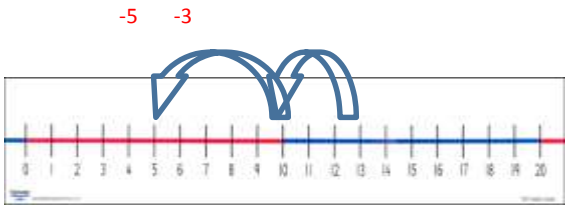
5. Use the relationship between addition and subtraction

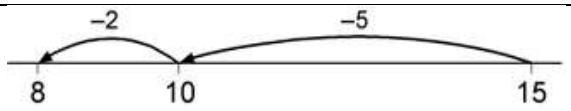
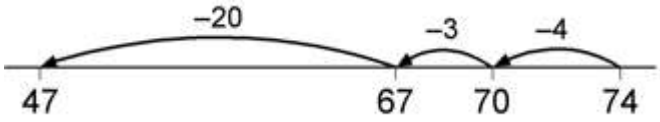
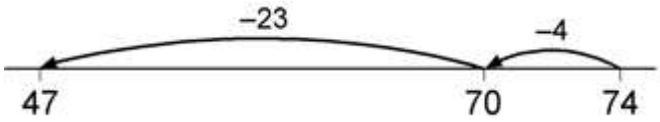
36 + 19 = 55 19 + 36 = 55
55 – 19 = 36 55 – 36 = 19

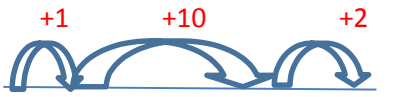
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEYARE NOT REPLACED BY WRITTEN METHODS.

WRITTEN METHODS FOR SUBTRACTION

Phase 1 Develop secure one-one correspondence and understanding of addition.	**Count accurately 0-10** • **Recognise and write numerals 1-10**  What is the number before 5? And after 5? Before 10? What is the number between 3 and 5? • What numbers are between 7 and 10? Count and add together sets of real objects and pictures 5 – 2 = 3 

Phase 2 Number line and 100 square	**Phase 2** To be able to subtract/take away one less on a number line. 8 – 2 = 6  To be able to subtract through 10 13 – 8 = 5  To be able to subtract 10 from any number up to 100 using a 100 square. 34-10=24 To be able to subtract multiples of 10 from any number up to 100 using a 100 square.	
Phase 3: **Using the empty number line**	**Phase 3** Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 – 7 = 8	

• The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.	 74 – 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined: 	

Phase 4 Continue using number lines to support subtraction as needed. Find a small difference by counting up • Read the number sentence. • Write the first number.	102 – 89 = 13  OR: 102 – 89 =13 102 - 89 90 (+ 1)	

• Write the second number underneath the first number making sure the digits are in the correct columns. • Partition each number into tens and units and write them underneath each other. • Subtract the units FIRST. • Next, subtract the tens. • Total the tens and units to get the answer.	100 (+ 10) 102 (+ 2) Expanded Written Method 89 – 57 = 89 → (80 + 9) - 57 → (50 + 7) 32 → 30 + 2 = 32	

Phase 5 **Column method of Subtraction** • In stage 5 children are taught the column method of subtraction. When teaching, encourage children link back to their place value understanding when subtracting. E.g. 70 – 20 in oppose to 7 – 2 in example 1 and 700 – 300 in oppose to 7-3 in example 2.	**Phase 5** Example 1: 74 – 27 6 14 7 4 - 2 7 4 7 Example 2: 741 – 367 6 13 11 7 4 1 - 3 6 7 3 7 4	Exchange

MULTIPLICATION

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS (ONGOING)

These are a selection of mental calculation strategies:

1. Doubling and halving

Applying the knowledge of doubles and halves to known facts.
e.g. 8 x 4 is double 4 x 4

2. Using multiplication facts

Tables should be taught regularly from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the week. All children should know these by heart by:
Year 2- 2 times table, 5 times table, 10 times table
Year 3- 2 times table, 3 times table, 4 times table, 5 times table, 6 times table, 10 times table
Year 4- Derive and recall all multiplication facts up to 10 x 10
Years 5 & 6- Derive and recall quickly all multiplication facts up to 10 x 10.

3. Using and applying multiplication facts

Children should be able to utilise their tables knowledge to derive other facts. e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc
Use closely related facts already known
13 x 11 = (13 x 10) + (13 x 1)
= 130 + 13
= 143

4. Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.
Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

5. Partitioning

23 x 4 = (20 x 4) + (3 x 4)
= 80 + 12
= 102

6. Use of factors

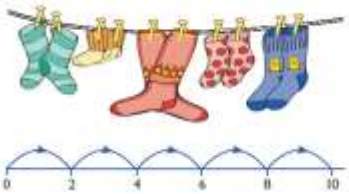
8 x 12 = 8 x 4 x 3

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

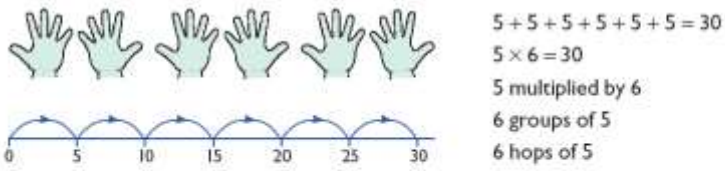
WRITTEN METHODS FOR MULTIPLICATION OF WHOLE NUMBERS

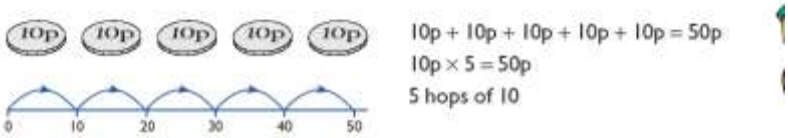
Phase 1:

Hands on experiences
Initially children put objects into groups/ sets

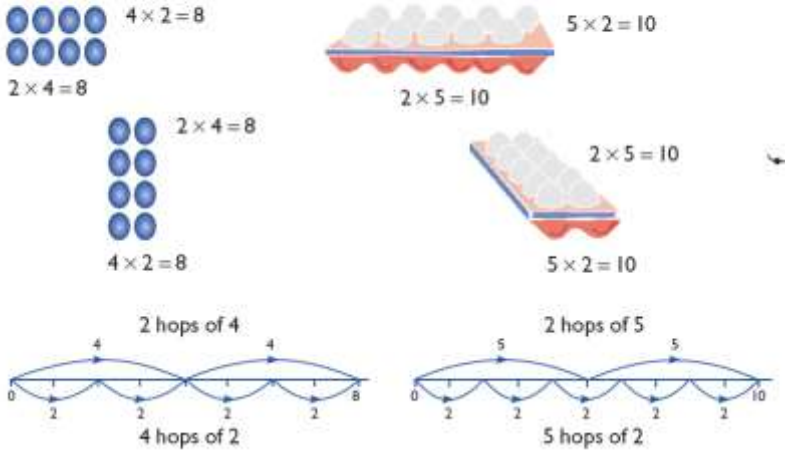


Solve multiplication through repeated addition.





To use arrays to solve multiplication problems.



To multiply by 4
17x4=
17x2x2= 34x2 =68

To multiply by 5
14x5=
(14x10)/2= 140/2 =70

Children begin to identify patterns within multiplications (e.g. number always ends with a 0, 2, 4, 6 or 8 in the 2X table) and between multiplications (4X table is double the 2X table).

Phase 2: **Mental multiplication using partitioning** • Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and units to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the units can be multiplied first but it is more common to start with the tens.	**Phase 2** 43 40 + 3 ↓ ↓ ×6 240 + 18 = 258 Also record mental multiplication using partitioning: 14 x 3 = (10 + 4) x 3 = (10 x 3) + (4 x 3) = 30 +12 = 42	**Key Vocab** Product, multiplication, array, partition
Phase 3: **The grid method** It is better to place the number with the most digits in the left hand column of the grid so that it is easier to add the partial products. We then progress on to the 2 digit by 2 digit multiplication using the grid. Again the larger number should be placed on the left to facilitate the addition stage.	**Grid method** To multiply 2 digit number by a 1 digit number using the grid method. 38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266 × | 7 30 | 210 8 | 56 | 266 56 x 27 is approximately 60 x 30 = 1800 × | 20 | 7 | 50 | 1000 | 350 | 1350 6 | 120 | 42 | 162 | | | 1512 | | | 1	As above Grid method, Partition Multiplication Column

Phase 4: **Short multiplication** • The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 4.	Phase 4 3 8 x 7 266 5 The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three digit number mentally before they reach this stage.	Carry
Phase 5: **Long Multiplication** • In this phase children are taught efficient methods for 2 digit by 2 digit multiplication and beyond. Make links with children's existing understanding of place value to ensure that they understand the need for a place value holder in the form of the '0'. Discuss the 2nd stage of the multiplication as 10 x 8 not 1 x 8 When understanding is secure, progress children onto 3 by 3 digit and 3 by 2 digit multiplication.	**Phase 5** 3 8 x 1 7 2 65 6 3 8 0 6 4 6 1	

DIVISION

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS (ONGOING)

These are a selection of mental calculation strategies:

1. Doubling and halving

Knowing that halving is dividing by 2

2. Deriving and recalling division facts

Tables should be taught regularly from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the week. All children should know these by heart by:
Year 2- 2 division table,5 times table,10 times table
Year 3- 2 times table,3 times table,4 times table,5 times table,6 times table,10 times table
Year 4- Derive and recall division facts for all tables up to 10 x 10
Year 5 & 6 - Derive and recall quickly division facts for all tables up to 10 x 10

3. Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
21 ÷ 7 = 3 so 210 ÷ 7 = 30, 210 ÷ 3 = 70, 210 ÷ 70 = 3 etc

4. Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.
Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

5. Use of factors

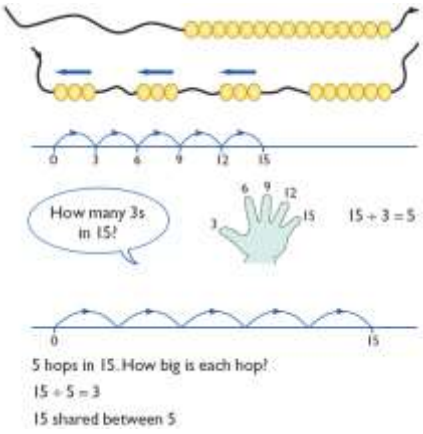
378 ÷ 21 378 ÷ 3 = 126 378 ÷ 21 = 18
126 ÷ 7 = 18

6. Use related facts

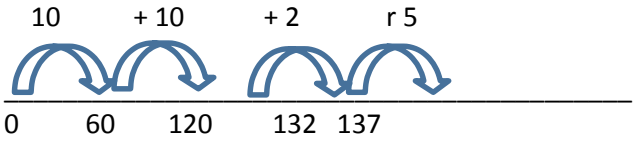
Given that 1.4 x 1.1 = 1.54
What is 1.54 ÷ 1.4, or 1.54 ÷ 1.1?

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Written methods for division

Phase 1 To divide on a number line initially with no remainders and later with remainders.	To half numbers to 10/20 – practically and mentally. To be able to divide practically by sharing. 

Phase 2: **Mental division using partitioning** • Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient. • Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing	**Phase 2** One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining units, then divide each part separately. 84 ÷ 7: 84 70 + 14 ÷ 7 10 + 2 = 12 In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.	

the correspondence to mental methods of multiplication, have not in the past been given enough attention. • Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.		
Phase 3: **Chunking on a number line and 'Expanded' method for HTU ÷ U** • This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. • As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?' • Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same. • This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several	**Phase 3** To divide on a number line by chunking. 137 / 6 = 22 r 5  97 / 9 10 r 7 9)97 -90 (9 x 10) 7 Answer: 10 r 7 196 / 6 32 r 4 6)196 -60 (6 x 10) 136 60 (6 x 10) 76 60 (6 x 10) 16 12 (6 x 2) 4 Answer: 32 r 4 At this stage children can also be taught the short division method: e.g. 96 / 5 1 9 r 1 5)9 46	Divisor Dividend Quotient Chunking

chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • Chunking is useful for reminding children of the link between division and repeated subtraction. • However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.		
Phase 4: The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend. • Estimating has two purposes when doing a division: – to help to choose a starting point for the division; – to check the answer after the calculation.	To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4. 196 / 6 32 r 4 6)196 -180 (6 x 30) 16 -12 (6 x 2) 4 Answer: 32 r 4 The quotient (32) lies between 30 and 40 as predicted	As above

• Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.		
Phase 5: **Long division** The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 units forming the answer are recorded above the line, as in the second recording.	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 560 / 24 23 r 8 24) 560 - 480 (24 x 20) 80 - 72 (24 x 3) 8 Answer: 23 r 8	As above